

5. 162 / 2b

b) I) $x^2 + 12 = 28 \quad | -12$

$$x^2 = 16$$

$$x_{1/2} = \pm 4$$

$$\mathcal{L} = \{ -4; 4 \}$$

II) $2x^2 - 7 = 3 \quad | +7$

$$2x^2 = 10 \quad | :2$$

$$x^2 = 5$$

$$x_{1/2} = \pm \sqrt{5}$$

$$\mathcal{L} = \{ -\sqrt{5}; \sqrt{5} \}$$

III) $12 - 3x^2 = -24 \quad | +24$

$$36 - 3x^2 = 0 \quad | +3x^2$$

$$36 = 3x^2 \quad | :3$$

$$12 = x^2$$

$$\Rightarrow x_{1/2} = \pm \sqrt{12}$$

$$\mathcal{L} = \{ -\sqrt{12}; \sqrt{12} \}$$

IV) $4 - x^2 = 5 \quad | -4$

$$-x^2 = 1 \quad | \cdot (-1); +x^2$$

$$x^2 = -1$$

$$\Rightarrow \mathcal{L} = \emptyset$$

Übungsblatt

1) d) $x^2 = 23$

2 Lsg.

$$x_{1/2} = \pm \sqrt{23}$$

$$\mathcal{L} = \{ -\sqrt{23}; \sqrt{23} \}$$

e) $x^2 = \frac{2}{3}$

2 Lsg.

$$\mathcal{L} = \{ -\sqrt{\frac{2}{3}}; \sqrt{\frac{2}{3}} \}$$

f) $5x^2 = 2$

$$x^2 = \frac{2}{5}$$

$$\mathcal{L} = \{ -\sqrt{\frac{2}{5}}; \sqrt{\frac{2}{5}} \}$$

g) $9x^2 = 12$

$$x^2 = \frac{12}{9} = \frac{4}{3}$$

$$\mathcal{L} = \{ -\sqrt{\frac{4}{3}}; \sqrt{\frac{4}{3}} \}$$

Übung 3

a) $2x^2 - 20x + 42$

$$D = b^2 - 4 \cdot a \cdot c$$

$$D = (-20)^2 - 4 \cdot (2) \cdot (42)$$

$$D = 400 - 336 = 64 > 0 \Rightarrow 2 \text{ Lösungen}$$

$$x_{1/2} = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \cdot (2) \cdot (42)}}{2 \cdot (2)}$$

$$= \frac{20 \pm \sqrt{36}}{4} = \frac{20 \pm 6}{4}$$

$$\Rightarrow x_1 = \frac{20+6}{4} = 6,5 \quad x_2 = \frac{20-6}{4} = 3,5 \Rightarrow \mathbb{L} = \{3,5; 6,5\}$$

b) $-3x^2 + 18x - 75 = 0$

$$D = 18^2 - 4 \cdot (-3) \cdot (-75)$$

$$D = 324 - 900$$

$$D = -576 \quad \text{keine Lösung}$$

$$\Rightarrow \mathbb{L} = \emptyset$$

c) $8 \cdot (x+3)(x-3) - 36 \cdot (2x-5) = 0$

$$8 \cdot (x^2 - 9) - 72x + 180 = 0$$

$$8x^2 - 72 - 72x + 180 = 0$$

$$8x^2 - 72x + 108 = 0$$

$$D = (-72)^2 - 4 \cdot 8 \cdot 108$$

$$5184 - 3456 = 1728$$

$$\Rightarrow 2 \text{ Lösungen}$$

$$x_{1/2} = \frac{-(-72) \pm \sqrt{1728}}{2 \cdot 8} = \frac{+72 \pm \sqrt{4 \cdot 4 \cdot 4 \cdot 3 \cdot 9}}{28}$$

$$= \frac{72 \pm \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot \sqrt{3}}{16}}{16} = \frac{72 \pm 24\sqrt{3}}{16}$$

$$= \frac{9 \pm 3\sqrt{3}}{2} \Rightarrow x_1 = \frac{9 + 3\sqrt{3}}{2} \quad x_2 = \frac{9 - 3\sqrt{3}}{2}$$

$$\Rightarrow \mathbb{L} = \left\{ \frac{9 + 3\sqrt{3}}{2}; \frac{9 - 3\sqrt{3}}{2} \right\}$$

$$\begin{aligned}
 d) \quad & d) \quad x(2x-5) = -1,5 \\
 & 2x^2 - 5x = -1,5 \quad | +1,5 \\
 & 2x^2 - 5x + 1,5 = 0 \\
 & D = (-5)^2 - 4 \cdot 2 \cdot 1,5 = 25 - 12 = 13 \\
 & x_{1/2} = \frac{-(-5) \pm \sqrt{13}}{2 \cdot 2} = \frac{5 \pm \sqrt{13}}{4} \\
 & L = \left\{ \frac{5 + \sqrt{13}}{4}; \frac{5 - \sqrt{13}}{4} \right\}
 \end{aligned}$$

Übung 4

$$\begin{aligned}
 a) \quad & D = (-1)^2 - 4 \cdot 1 \cdot q \\
 & D = 1 - 4q \\
 & D \geq 0 \Leftrightarrow 1 - 4q \geq 0 \\
 & \quad \cdot 1 \geq 4q \quad | :4 \\
 & \quad + \frac{1}{4} \geq q
 \end{aligned}$$

Zwei Lösungen für $q < \frac{1}{4}$

$$\begin{aligned}
 x_{1/2} &= \frac{-(-1) \pm \sqrt{1-4q}}{2 \cdot 1} = \frac{1 \pm \sqrt{1-4q}}{2} \\
 L &= \left\{ \frac{1 + \sqrt{1-4q}}{2}; \frac{1 - \sqrt{1-4q}}{2} \right\}
 \end{aligned}$$

eine Lösung für $q = \frac{1}{4} \Rightarrow D = 0$

$$\begin{aligned}
 x_{1/2} &= \frac{-(-1) \pm \sqrt{0}}{2} = \frac{1}{2} \\
 \Rightarrow L &= \left\{ \frac{1}{2} \right\}
 \end{aligned}$$

keine Lösung für $q > \frac{1}{4} \Rightarrow L = \emptyset$

$$b) \quad x^2 - 0,25q - 7x = 0$$

$$D = (-7)^2 - 4 \cdot 1 \cdot (-0,25q) = 49 + q$$

$$D = 49 + q \geq 0 \Rightarrow q \geq -49$$

2 Lösungen für $q > -49$

$$x_{1/2} = \frac{-(-7) \pm \sqrt{49+q}}{2 \cdot 1} \Rightarrow L = \left\{ \frac{7 + \sqrt{49+q}}{2}; \frac{7 - \sqrt{49+q}}{2} \right\}$$

eine Lösung für $q = -49$

$$x_{1/2} = \frac{-7 \pm 0}{2} = \frac{7}{2}$$

$$\mathbb{L} = \left\{ \frac{7}{2} \right\}$$

keine Lösung für $q < -49$

c) $5x^2 - 7x = 49q$

$$5x^2 - 7x - 49q = 0$$

$$D = (-7)^2 - 4 \cdot 5 \cdot (-49)$$

$$D = 49 + 980q$$

Durch 5

$$49 + 980q = 0$$

$$980q = -49 \quad | :980$$

$$q = -\frac{1}{20}$$

2 Lösungen für $q > -\frac{1}{20}$

1 Lösung für $q = -\frac{1}{20}$

0 Lösungen für $q < -\frac{1}{20} \Rightarrow \mathbb{L} = \emptyset$

$$x_{1/2} = \frac{-(-7) \pm \sqrt{49 + 980q}}{2 \cdot 5} = \frac{7 \pm \sqrt{1 + 20q}}{10}$$

$$x_1 = \frac{7 + \sqrt{1 + 20q}}{10}; \quad x_2 = \frac{7 - \sqrt{1 + 20q}}{10}$$

$$\mathbb{L} = \{ \dots \}$$

$$x_{1/2} = \frac{-7 \pm 0}{10} \Rightarrow \mathbb{L} = \left\{ -\frac{7}{10} \right\}$$

d) $qx^2 + 1 = 0,8 \Leftrightarrow qx^2 + 0,2 = 0$

$$D = -4q \cdot 0,2 = 0$$

$$-q \cdot 0,8 = 0$$

$$q = 0$$

2 Lösungen: $q < 0$ 1 Lsg $q = 0$

$$x_{1/2} = \frac{-0 \pm \sqrt{-0,8q}}{2} \Rightarrow \mathbb{L} = \left\{ \frac{\sqrt{-0,8q}}{2}; -\frac{\sqrt{-0,8q}}{2} \right\}$$

$$x = 0 \Rightarrow \mathbb{L} = \{0\}$$

Typische Aufgabenstellungen

(1)

$$a) \sqrt{(-4)^2 - 4 \cdot 1 \cdot m} = D$$

$$16 - 4m = D$$

$$D = 0 : 1 \text{ Lösung}$$

$$16 - 4m = 0$$

$$16 = 4m \quad | :4$$

$$4 = m$$

$$D > 0 : 2 \text{ Lösungen für } m < 4$$

$$16 - 4m > 0$$

$$16 > 4m \quad | :4$$

$$4 > m$$

$$\Rightarrow \text{für } m < 4$$

$$\text{keine Lösung für } m > 4$$

$$\text{|| für } D = 0:$$

$$x_{1/2} = \frac{-(-4) \pm 0}{2} = \frac{4}{2} = 2 \Rightarrow \mathbb{L} = \{2\}$$

$$\text{|| für } D < 0$$

$$\text{|| für } D > 0:$$

$$x_{1/2} = \frac{-(-4) \pm \sqrt{16 - 4m}}{2} = \frac{4 \pm \sqrt{4(4-m)}}{2} =$$

$$= \frac{4 \pm 2\sqrt{4-m}}{2} = 1 \pm \sqrt{4-m}$$

$$\mathbb{L} = \{1 + \sqrt{4-m}; 1 - \sqrt{4-m}\}$$

$$b) D = (+m)^2 - 4 \cdot 1 \cdot 12$$

$$D = m^2 - 48$$

$$D > 0 \Leftrightarrow m^2 - 48 > 0$$

$$m^2 > 48$$

$$m > \pm 2\sqrt{12}$$

$$m > \pm 4\sqrt{3}$$

$$D = 0 \Leftrightarrow m = \pm 4\sqrt{3}$$

$$D < 0 \Leftrightarrow m < \pm 4\sqrt{3} \Rightarrow \mathbb{L} = \emptyset$$

ohne Lösungsmengen, da für $D > 0$, da hier Fallunterscheidung nötig

$$D > 0 \Leftrightarrow x_{1/2} = \frac{-m}{2} = -\frac{m}{2} \Rightarrow \mathbb{L} = \left\{-\frac{m}{2}\right\}$$

$$c) (x-m)^2 = x^2 - 2xm + m^2$$

$$D = (-2m)^2 - 4 \cdot 1 \cdot m^2$$

$$D = 4m^2 - 4m^2 = 0$$

$\Rightarrow$ es gibt nur eine Lösung für alle $m \in \mathbb{R}$:

$$x_{1/2} = \frac{-(-2m)}{2} = +m \Rightarrow \mathbb{L} = \{+m\}$$

$$d) (x+m)^2 = x^2 + 2xm + m^2$$

$$\Rightarrow D = (2m)^2 - 4 \cdot 1 \cdot m^2$$

$$D = 4m^2 - 4m^2 = 0$$

$\Rightarrow$ es gibt nur eine Lösung für alle $m \in \mathbb{R}$:

$$x_{1/2} = \frac{-2m}{2} = -m \Rightarrow \mathbb{L} = \{-m\}$$

$$e) D = 7^2 - 4 \cdot 1 \cdot m$$

$$D = 49 - 4m$$

$$D > 0 \Leftrightarrow 49 - 4m > 0$$

$$49 > 4m \quad | :4$$

$$12,25 > m$$

$$x_{1/2} = \frac{-7 \pm \sqrt{49 - 4m}}{2}$$

$$\mathbb{L} = \left\{ \frac{-7 + \sqrt{49 - 4m}}{2} ; \frac{-7 - \sqrt{49 - 4m}}{2} \right\}$$

$$D = 0 \Leftrightarrow 49 - 4m = 0 \Rightarrow m = 12,25$$

$$\Rightarrow x_{1/2} = \frac{-7}{2} = -3,5 \Rightarrow \mathbb{L} = \{-3,5\}$$

$$m > 12,25 \Rightarrow \mathbb{L} = \emptyset$$

$$(2) a) X \in \mathbb{N} \Rightarrow \mathbb{D} = \mathbb{N}$$

$$X^2 - 3X = 130$$

$$X^2 - 3X - 130 = 0$$

$$x_{1/2} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-130)}}{2 \cdot 1} =$$

$$= \frac{3 \pm \sqrt{529}}{2} = \frac{3 \pm 23}{2}$$

$$x_1 = \frac{3+23}{2} = \frac{26}{2} = 13$$

$$x_2 = \frac{3-23}{2} = -\frac{20}{2} = -10 \notin \mathbb{N} \Rightarrow x_2 \notin \mathbb{D}$$

$$\Rightarrow \mathbb{L} = \{13\}$$

$$b) X \in \mathbb{N} \Rightarrow \mathbb{D} = \mathbb{N}$$

$$X^2 + X = 132$$

$$X^2 + X - 132 = 0$$

$$x_{1/2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-132)}}{2 \cdot 1} =$$

$$= \frac{-1 \pm \sqrt{529}}{2} = \frac{-1 \pm 23}{2}$$

$$x_1 = \frac{-1+23}{2} = \frac{22}{2} = 11$$

$$x_2 = \frac{-1-23}{2} = -\frac{24}{2} = -12 \notin \mathbb{N} \Rightarrow x_2 \notin \mathbb{D}$$

$$\Rightarrow \mathbb{L} = \{11\}$$

$$c) X \cdot (X+1) = 812 \quad X \in \mathbb{N} \Rightarrow \mathbb{D} = \mathbb{N}$$

$$X^2 + X = 812$$

$$X^2 + X - 812 = 0$$

$$x_{1/2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-812)}}{2 \cdot 1} =$$

$$= \frac{-1 \pm \sqrt{3249}}{2} = \frac{-1 \pm 57}{2}$$

$$x_1 = \frac{-1+57}{2} = \frac{56}{2} = 28$$

$$x_2 = \frac{-1-57}{2} = -\frac{58}{2} = -29 \notin \mathbb{N} \Rightarrow x_2 \notin \mathbb{D}$$

$$\Rightarrow L = \{2, 8\}$$

$$(4) \text{ allg.: } ax^2 + bx + c = 0$$

$$a) x_1 = -2 \quad x_2 = 3$$

$$\Rightarrow 0 \cdot (x+2) \cdot (x-3) = 0$$

$$a(x^2 - 3x + 2x - 6) = 0$$

$$a(x^2 - x - 6) = 0$$

$$\text{mit } a = 2$$

$$2x^2 - 2x - 12 = 0$$

$$b) x_1 = -7 \quad x_2 = 3$$

$$(x+7) \cdot (x-3) = 0$$

$$x^2 + 7x - 3x - 21 = 0$$

$$x^2 + 4x - 21 = 0$$

$$c) x_1 = -3,5 \quad x_2 = 1,5$$

$$(x+3,5) \cdot (x-1,5) = 0$$

$$x^2 + 3,5x - 1,5x - 5,25 = 0$$

$$x^2 + 2x - 5,25 = 0$$

$$c = -2$$

$$\Rightarrow +5,25 \cdot c' = 2$$

$$c' = +\frac{2}{5,25}$$

$$\frac{+2x^2}{5,25} + \frac{+4x}{5,25} - 2 = 0$$